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The mean transverse momentum of secondaries from cosmic-ray interactions in the region of 2×10^5 GeV

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Abstract. The variation with separation of the rate of pairs of cosmic-ray muons of energies above 1000 GeV has been measured at a mean angle to the vertical of 45° by Coats *et al.* The variation is most sensitive to the mean transverse momentum of the parent pions in the initial nucleon–air nucleus interactions and the value determined from comparison with a theoretical analysis is $\langle p_t \rangle = 0{\cdot}60 \pm 0{\cdot}05$ GeV/c, the median primary energy in question being approximately 2×10^5 GeV. The form of the distribution in transverse momentum giving a best fit to the experimental data is that proposed by Elbert *et al.*

The value found for $\langle p_t \rangle$ lends confirmation to the slow increase in this quantity with increasing interaction energy suggested by previous work.

1. Introduction

The examination of the transverse momentum of the secondaries from high-energy interactions is important in view of the invariance of this quantity with respect to coordinate system. Previous studies (surveyed by de Beer *et al.* 1968) using accelerators at energies below a few tens of GeV and cosmic rays above, have shown that the mean transverse momentum ($\langle p_t \rangle$) of the pions generated in nucleon–light nucleus interactions increases very slowly with energy, reaching approximately 0·5 GeV/c for energies approaching 10^5 GeV. The present work, which refers to measurements of the lateral distribution of the secondary muons, enables $\langle p_t \rangle$ to be determined at even higher interaction energies.

The experimental data come from the University of Utah detector (Bergeson *et al.* 1967) a large device comprising directional Čerenkov detectors, cylindrical spark counters and solid iron magnets, which is located in a mine under the rugged Wasatch mountains. The terrain is such that cosmic-ray muons, of a variety of sea-level threshold energies, can be selected at a fixed zenith angle by looking at various azimuth angles. In addition to single muons penetrating the device many examples of multiple muons have been observed, these being the residue of extensive air showers whose electron and hadron components have been absorbed in the overlying rock.

Results on the density spectrum of muons taken with the main detector have already been published by Porter and Stenerson (1969) and calculations aimed at interpreting the data have been reported by Adcock *et al.* (1969), the conclusion drawn being that there was no evidence for a value of $\langle p_t \rangle$ significantly above the 0·4 GeV/c which relates to lower primary energies. Recent new data, reported in the previous paper by Coats *et al.*(1970), allow the problem to be examined in more detail and they form the basis of the present work. The new data come principally from outrigger detectors which are operated in an adjacent tunnel and which enable muons separated by as much as 50 m from those in the main array to be detected. Such large separations not only allow $\langle p_t \rangle$ to be determined to greater precision than hitherto but offer the eventual possibility of studying the shape of the p_t distribution at large p_t.

2. Experimental data

The basic data and their collection are described in detail in the accompanying paper and only a brief description will be given here. The data were selected using a pattern recognition computer program for parallel muons in the main array, and the coincident muons in the outrigger detectors were analysed individually by hand plotting. 5507 pairs of parallel penetrating muons were recorded during the period from January to June 1969 which fell into the accepted range of zenith angle and depth of rock traversed: 40 to 70° and 1900 to 4000 hg cm^{-2} (muon threshold energies in the range 700–2400 GeV, assuming that the energy loss coefficient b is $4 \cdot 0 \times 10^{-6} g^{-1} cm^2$). The separation between each pair of muons was calculated and the events were sorted into cells of zenith angle and depth of rock traversed. The aperture and triggering requirements of the detector were folded out to yield the counting rates for an equivalent pair of 1 m^2 detectors as a function of their separation —the 'decoherence curve'—for the various cells of angle and depth. Insofar as the emphasis in the present work is on the derivation of one physical parameter the data for the various cells are combined to give the decoherence curve at a specific zenith angle (45°) and threshold energy (1050 GeV). The manner of combination comes from theoretical prediction and a rough check on its validity was given in the previous paper.

The derivation of physical information from the variation of rate of multiples (both 2's and higher values) with angle and depth will be discussed in detail in a later paper.

3. Theoretical analysis

3.1. *General comments*

As with all results of this type the method of analysis is to calculate the expected results for a variety of values of parameters concerning the primary radiation and high energy interaction properties and to accept those giving a best fit to the experimental data.

The number of parameters affecting the various cosmic-ray measurements is great but the lateral distribution of muons, and the closely linked decoherence curve, is notable in that it depends to a large extent on only one parameter—the distribution of transverse momentum in the interaction from which the immediate parents of the muons come and the present work concerns the derivation of this parameter.

3.2. *The preferred interaction model*

In previous work it has been shown that the simple model used by de Beer *et al.* (1966) gives an adequate description of a number of cosmic-ray phenomena. This model is therefore adopted here as the 'preferred' model and accurate calculations have been made using it; other variants are used later to examine their sensitivity.

Briefly the model assumes that:

(i) Pions are the main source of muons and that their total number depends on primary energy E_p (in GeV) according to $n_s = 2 \cdot 7 \times 2^{1/4} (KE_p)^{1/4}$ where K, the inelasticity, is equal to one half for nucleon interactions and unity for pion interactions. Equal numbers of π^+, π^- and π^0 mesons are assumed.

(ii) The mean free paths for interactions are $\lambda_p = \lambda_n = 80$ g cm^{-2}, $\lambda_\pi = 120$ g cm^{-2}.

(iii) The pions have an energy distribution in the laboratory system given by the empirical relation due to Cocconi *et al.* (1961—to be referred to as CKP) in which half the pions are emitted backward in the centre-of-mass system, and thus have low energy in the laboratory (L-) system, and half are emitted forward with an energy spectrum in the L-system given by

$$n(E) = \frac{A}{T} \exp\left(-\frac{E}{T}\right)$$

T being the mean energy.

(iv) The distribution of transverse momentum p_t of the produced pions is given by the CKP relation

$$f(p_t) = \frac{p_t}{p_0^2} \exp\left(-\frac{p_t}{p_0}\right)$$

where $\langle p_t \rangle = 2p_0$ the mean value taken initially as $0{\cdot}4$ GeV/c is regarded as a variable.

(v) The primary cosmic-ray particles are mainly protons.

At this stage it is necessary to comment on the possibility of a new process—the X-process—for which the Utah group (Bergeson *et al.* 1967) have presented evidence. In this it is suggested that a large fraction of the muons having energy above 1000 GeV have come from the decay of an intermediate particle having lifetime much less than that of the pion or the kaon. This suggestion comes from the observation of an angular distribution of single muons which is flatter than is predicted by the normal pion (or kaon) decay mechanism. In the present work such a mechanism is not considered as a possible source of detected muons since the object is to examine the predictions of a conservative interaction model. Such a procedure does not necessarily lead to inconsistencies because the contribution of X-process muons to the detected showers of muons may not be great in view of the bias towards high primary particle interaction points which means that the flux of muons from ordinary pion decay is enhanced. An analysis of the relevance of the X-process to the multiple muon data will also be given in the later paper.

3.3. *Variants of the model*

3.3.1. *Transverse momentum distribution.* As mentioned in § 1 a study of the form of the p_t distribution is an eventual possibility from decoherence measurements, and alternative forms have been tried. These other forms have been derived, in the main, from nucleon–nucleon or pion–nucleon interaction experiments rather than nucleon–air nucleus interactions as required in the present application, so that, owing to secondary cascading within the air-nucleus, such forms may not in fact be directly applicable. However, no precise accelerator results for nucleon–light nucleus interactions are available from which to take other trial p_t distributions and so these forms have been adopted. They are:

$$N(p_t) = \frac{1}{1{\cdot}33 p_0} \left(\frac{p_t}{p_0}\right)^{3/2} \exp\left(-\frac{p_t}{p_0}\right)$$

Elbert *et al.* (1968) based on 25 GeV/c π^-p interactions.

with $p_0 = 0{\cdot}16$.

$$N(p_t) = \frac{2p_t}{p_0^2} \exp\left(-\frac{p_t}{p_0}\right)^2$$

Aly *et al.* (1964) based on both machine data (π^-p, p–n) and cosmic-ray jet results.

with $p_0 = 0{\cdot}45$.

Another distribution has been measured recently, that due to Ratner *et al.* (1967); $N(p_t) \propto \exp(-\alpha p_t^2)$, for $0{\cdot}4 < p_t < 1{\cdot}3$ GeV/c, this being derived from 12·5 GeV pp reactions. However, it differs from that of Elbert *et al.* by a negligible amount over the range in question and has thus not been taken as a variant. The various distributions are shown in figure 1.

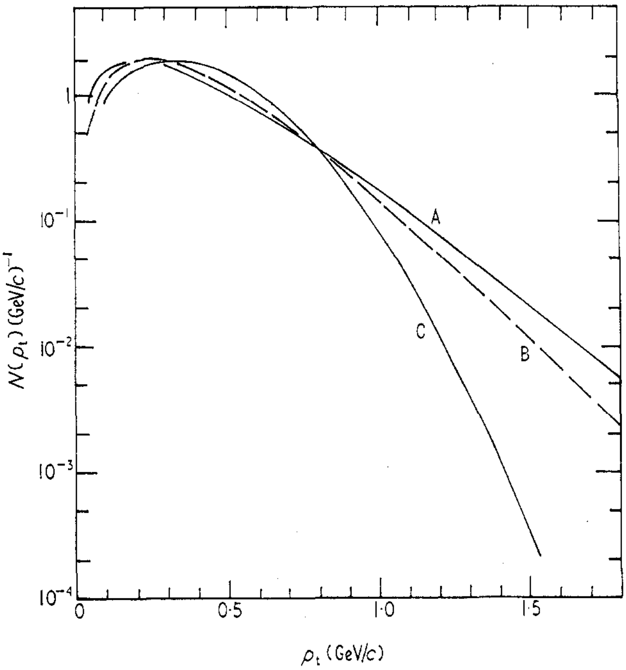

Figure 1. Trial distributions of transverse momentum. The mean value is 0·4 GeV/c in each case. A—CKP, $N(p_t) = p_t/p_0^2 \exp(-p_t/p_0)$. B—Elbert *et al.*, $N(p_t) = 1/1{\cdot}33 p_0 (p_t/p_0)^{3/2} \exp(-p_t/p_0)$. C—Aly *et al.*, $N(p_t) = 2(p_t/p_0^2) \times \exp(-p_t/p_0)^2$.

3.3.2. *Primary mass composition.* If the primary cosmic rays are predominantly heavy nuclei rather than protons then the effective pion multiplicity in the early interactions is increased and the cascade develops more rapidly (a similar situation arises if the multiplicity increases more rapidly than $E_p^{1/4}$). In addition to calculations for a spectrum composed of protons only, an analysis has therefore also been made for a primary spectrum in which heavy nuclei play an important part. This alternative spectrum is one in which the composition is assumed to be constant up to 10^{15} eV/nucleus and identical with that directly measured in the region of 10^{10} eV; above 10^{15} eV it is assumed that there is an increase in mean primary mass as the low-mass particles are lost through inefficient trapping in the galactic magnetic field. (The two alternative primary spectra are discussed by de Beer *et al.* 1969).

4. Calculated lateral distributions and decoherence curves

4.1. *Average lateral distributions*

As remarked in § 3, the basic calculations are made for primary protons and the conservative interaction model. The mode of calculation is to determine the expected ground-level lateral distribution of muons from primary interactions occurring at a unique level for a variety of level heights and interaction energies. These distributions are then weighted appropriately over the production levels to give the resultant average lateral distribution at ground level. Such distributions are shown in figure 2

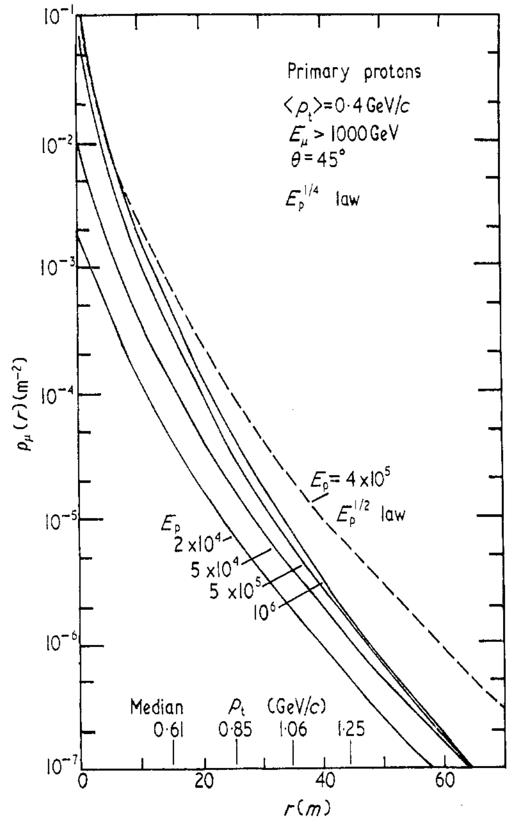

Figure 2. Average lateral distribution of muons produced by the interactions of primary protons. E_p denotes the primary proton energy.

for various primary proton energies. Shown on the figure are the median p_t values for various radial distances, i.e. the transverse momenta such that half the detected muons at that particular distance have a lower and half a higher p_t value, for $E_p = 2\times10^5$ GeV. An interesting feature is the rather slow increase in median p_t with increasing distance, a fact that comes from the median height of origin of the muons increasing and the mean muon energy decreasing as r goes up.

The shape of the lateral distribution can be understood with reference to the distributions that can be calculated under simplifying assumptions for production of first-generation pions by a nucleon of energy E_p at a unique height h.

In first approximation, assuming that all the pions have the same energy E_π and, similarly, that the resulting muons are mono-energetic ($E_\mu = E_\pi/1{\cdot}3$) the density at a distance r is given by

$$\rho_\mu(r, h, E_\mu) \propto \frac{E_\mu}{h^2 p_0{}^2} \exp\left(-\frac{1{\cdot}3 r E_\mu}{h p_0}\right)$$

i.e.

$$\propto \frac{1}{E_\mu r_0{}^2} \exp\left(-\frac{r}{r_0}\right)$$

where

$$r_0 = \frac{hp_0}{1 \cdot 3 E_\mu}.$$

In second approximation, again for a constant value of h but now using the CKP pion energy spectrum and integrating over muon energies above E_μ, we have

$$\rho_\mu(r, h, > E_\mu) \propto \frac{1}{h^2 p_0{}^2 T}\left(\frac{1 \cdot 3 E_\mu}{\alpha} + \frac{1}{\alpha^2}\right) \exp(-1 \cdot 3 \alpha E_\mu)$$

where

$$\alpha = \frac{r}{hp_0} + \frac{1}{T}$$

T being the mean pion energy for the nucleon energy E_{p}.

For $r/hp_0 \gg 1/T$ the expression reduces to

$$\rho_\mu(r, h, > E_\mu) \propto \frac{1}{Trr_0} \exp\left(-\frac{r}{r_0}\right).$$

With more accurate calculations, allowing for later generations of pions and summing over h and the primary spectrum, the concave nature of the lateral distribution (on the logarithmic–linear plot) is somewhat enhanced. Furthermore, the increasing importance of the later generations with increasing primary energy explains the slow change of shape with E_{p}.

4.2. *Correction for fluctuations in muon energy loss*

So far the discussion has concerned the lateral distributions expected for different muon threshold energies whereas what is measured is the frequencies of muons observed underground. Use of the range–energy relation alone leads to significant errors because of the well-known effect of fluctuations in energy loss which give rise to enhanced underground intensities. The enhancement factor increases with slope of the muon energy spectrum and is thus largest at large radial distances where the slope is greatest. For example, for the conditions of figure 2 the integral exponent at r = 30 m is $\gamma \sim 4 \cdot 4$ and the enhancement factor is approximately 1·7.

Calculations have been made for the energies of muons contributing to the experimental data and the result of applying the correction is shown in figure 3. The effect is an increase in width of 9%.

4.3. *Decoherence curves*

In the present application it is the decoherence curves, rather than the individual lateral distributions, that are measured. The predicted decoherence curve for the standard conditions has been derived by summing the probabilities of a single muon passing through each of 1 m^2 areas, separated by a distance x, for showers whose axes are distributed uniformly throughout the vicinity of the detectors and which have the appropriate primary energy spectrum.

The relationship of the decoherence rate $R(x)$ to $\rho_\mu(r)$ can be seen from an approximate analysis in which it is assumed that the lateral distribution is exponential with a mean radius r_0 independent of primary energy (for fixed muon threshold energy and angle).

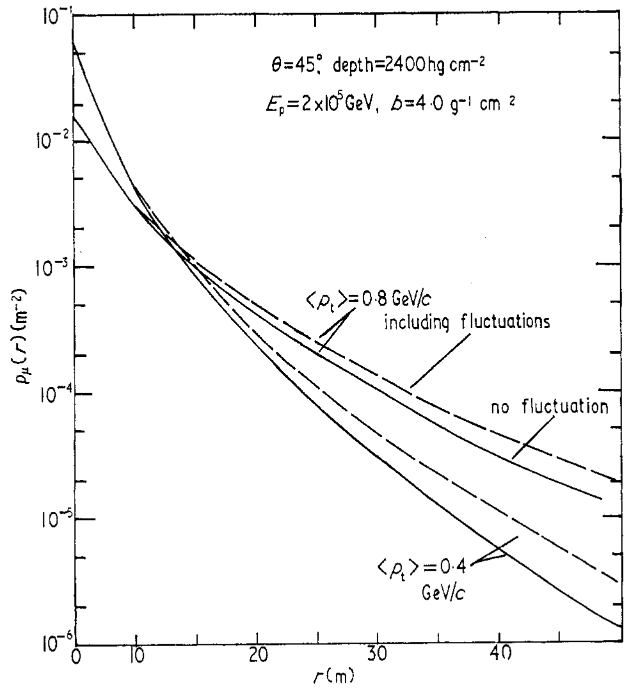

Figure 3. Average lateral distribution of muons, showing the effect of fluctuations in muon energy loss.

It can be shown that

$$R(x) = \frac{\int N_\mu{}^2(E_\mathrm{p})}{16 r_0{}^4} j(E_\mathrm{p}) \mathrm{d}E_\mathrm{p} \left\{ -\frac{r_0 \pi}{2} \mathrm{H}_1^1\left(\mathrm{i}\frac{x}{r_0}\right) + x^2 \frac{\pi}{2} \mathrm{i} \mathrm{H}_0^1\left(\mathrm{i}\frac{x}{r_0}\right)\right\}$$

where the H are Hankel functions, $N(E_\mathrm{p})$ is the number of muons of energy above threshold produced by a primary of energy E_p and $j(E_\mathrm{p})$ is the primary intensity. For large values of x/r_0 this reduces to

$$R(x) \propto r_0{}^{-3} x^{1/2} \left(r_0 + \frac{x}{2}\right) \exp\left(-\frac{x}{r_0}\right)$$

i.e.

$$R(x) \propto x^{3/2} \exp\left(-\frac{x}{r_0}\right) \quad \text{for } x \gg r_0.$$

Thus the convexity of the $R(x)$ plot is explained and also the fact that its slope is less than that of $\rho_\mu(r)$.

The considerable sensitivity of $R(x)$ to r_0, and thus to the mean transverse momentum, is clearly seen from this expression.

Calculation of the expected decoherence curve has been made using the actual lateral distributions, and their slow variation with primary energy, (for example, r_0 is typically 3·0, 3·8 and 4·2 m for $E_\mathrm{p} = 5 \times 10^5$, 10^5 and 2×10^4 GeV respectively at $r = 10$ m for $\langle p_\mathrm{t} \rangle = 0{\cdot}4$ GeV/c) with the result shown in figure 4.

In point of fact the mean lateral distributions are not used directly because by so doing correlations in muon numbers would not be allowed for—the increase in lateral spread and decay probability with height means that the multiplicity of generated muons increases quite rapidly with height and large values of x are derived preferentially from these groups of coherent muons. Elementary decoherence curves are therefore derived for the various production levels and these are weighted and

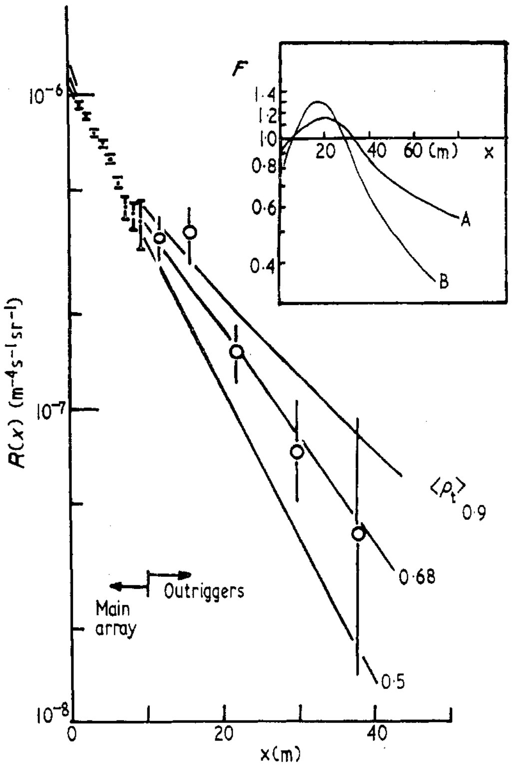

Figure 4. Expected decoherence curve and comparison with observation, (for $\theta = 45°$, $E_\mu > 1050$ GeV). The theoretical curves refer to the CKP p_t distribution; the curve marked $\langle p_t \rangle = 0{\cdot}68$ is the best fit to the experimental data from the main array (x values less than 10 m). The inset shows the ratio (F) of decoherence rate predicted by a particular distribution with $\langle p_t \rangle = 0{\cdot}68$ GeV/c to the rate predicted by the CKP distribution with $\langle p_t \rangle = 0{\cdot}68$ GeV/c, as a function of x. A, Elbert *et al.* (1968); B, Aly *et al.* (1964).

summed; the difference between this treatment and the more elementary one using the average lateral distribution of figure 2 amounts to an increase in $R(x)$ (for $\theta = 45°$, $E_\mu > 1000$ GeV) varying from 1·4 at $r = 10$ m to 2·65 at $r = 40$ m in the case where $\langle p_t \rangle = 0{\cdot}4$ GeV/c.

The theoretical curves given in figure 4 refer to three values of $\langle p_t \rangle$ for the CKP distribution and the inset shows the sensitivity of $R(x)$ to the form of $N(p_t)$. It is clear that given adequate statistical accuracy, particularly for x greater than about 30 m, then not only should a precise value of $\langle p_t \rangle$ be derivable but also the form of $N(p_t)$.

4.4. *Dependence of the decoherence curve on θ and E_μ*

Before proceeding to a comparison of theory and experiment for the overall decoherence curve, the origin of the adopted scaling laws will be explained; the result of the scaling was given in the previous paper and some measure of experimental justification was advanced.

In first approximation the width of the lateral distribution (i.e. the scale factor for radial distance) and thus the width of the decoherence curve is proportional to $\sec\theta/E_\mu$, the $\sec\theta$ coming from simple geometrical considerations and the E_μ from p_t arguments. Using the accurate treatment a slightly different result appears: width $\propto \sec^{1\cdot 3}\theta E_\mu{}^{-0\cdot 8}$ for the angular and energy ranges in question. The corresponding accurate intensity factor is $R(x) \propto \sec^{-0\cdot 8}\theta E_\mu{}^{-1\cdot 69}$. The exponent of $\sec\theta$ is greater than unity because the vertical depth in the atmosphere from which the muons come decreases as θ increases and the inclined distance to the production levels thus increases faster than $\sec\theta$; the exponent of energy differs from unity because the muon energy spectrum has a slope increasing with energy and the ratio of mean energy to threshold energy E_μ, therefore decreases with E_μ.

5. Comparison with experiment and conclusions

5.1. *Determination of $N(p_t)$ and $\langle p_t \rangle$ for the preferred model*

The complete, scaled data from the Utah experiment are given in figure 4, where comparison is made with theoretical curves for the CKP form of $N(p_t)$. The experimental points at separations (x) less than 10 m come from the main array and refer to the depth range 2000–4000 hg cm^{-2}. Those at larger distances are derived from events in which one of the muons is in the main array and the other is in an outrigger detector, the relevant depth range is 1900–3900 hg cm^{-2}.

The best fit value of $\langle p_t \rangle$ has been found using the χ^2 test and for two sets of data: that from the main array alone and that for the whole data including the outrigger events. The χ^2 values for the different forms of $N(p_t)$ tried are given in table 1.

Table 1.

Form of $N(p_t)$	Main array only			All data		
	Min χ^2	Significance	$\langle p_t \rangle$ (GeV/c)	Min χ^2	Significance	$\langle p_t \rangle$ (GeV/c)
Aly *et al.* (1964)	6·0	55%	0·40 ± 0·06	33	0·1%	0·59 ± 0·06
Elbert *et al.* (1968)	5·3	63%	0·58 ± 0·07	11·5	48%	0·60 ± 0·05
CKP (1961)	4·8	69%	0·68 +0·08 −0·05	12·5	40%	0·71 ± 0·09

The percentages under the heading 'Significance' represent the probability of observing at least the value of χ^2 quoted.

It can be seen that there is a rather wide range of possible mean values using the main array data alone but with the outrigger events included the range is reduced and, furthermore, one of the distributions—that due to Aly *et al.* (1964)—can be ruled out. The p_t distribution which fits the data best is that due to Elbert *et al.* (1968) although the CKP distribution has almost as high a probability of fitting.

5.2. *Sensitivity to mass composition of the primary particles*

It has already been mentioned that the nature of the mass composition of the primaries is uncertain. The calculations presented refer to primary protons but approximate calculations of the decoherence curve expected for a modulated composition have also been made. For a given energy per nucleus the lateral distribution of muons from a heavy nucleus is wider than that for a primary proton but because of the much lower energy per nucleon in the case of the heavy nucleus the yield of muons is much less; thus of those muons coming from heavy-nucleus interactions most come from rather higher primary nucleus energies and because of the rapid fall off in primary spectrum their influence on the total lateral distribution, and thus on the decoherence curve, is small. The result is that there is no significant change in the decoherence curve and in turn there is no uncertainty in the derived value of $\langle p_t \rangle$ because of lack of precise knowledge of the primary mass composition.

5.3. *Sensitivity to multiplicity law*

Of greater importance is the change which results if the multiplicity of secondaries rises more rapidly than $E_p^{1/4}$. Approximate calculations for an $E_p^{1/2}$ multiplicity law give lateral distributions wider by a factor approaching 50% and thus the value of $\langle p_t \rangle$ derived from a comparison of the observed and expected decoherence curves would be approximately 30% less, i.e. about 0·44 GeV/*c*. However, as has already been remarked other evidence tends to point against such a high multiplicity-law exponent. In fact the evidence from the Utah experiment itself (Adcock *et al.* 1969) argues against such a rapid increase of multiplicity.

5.4. *Comparison with other $\langle p_t \rangle$ estimates*

Figure 5 shows a summary of $\langle p_t \rangle$ values taken from a variety of other experiments. Some refer to charged and some to neutral pions (a distinction is made in the figure) and the source of the measurement varies; with increasing particle energy the data came from: bubble chamber studies using machine-accelerated particles, cloud chamber observations of cosmic rays and finally a variety of emulsion studies of cosmic rays.

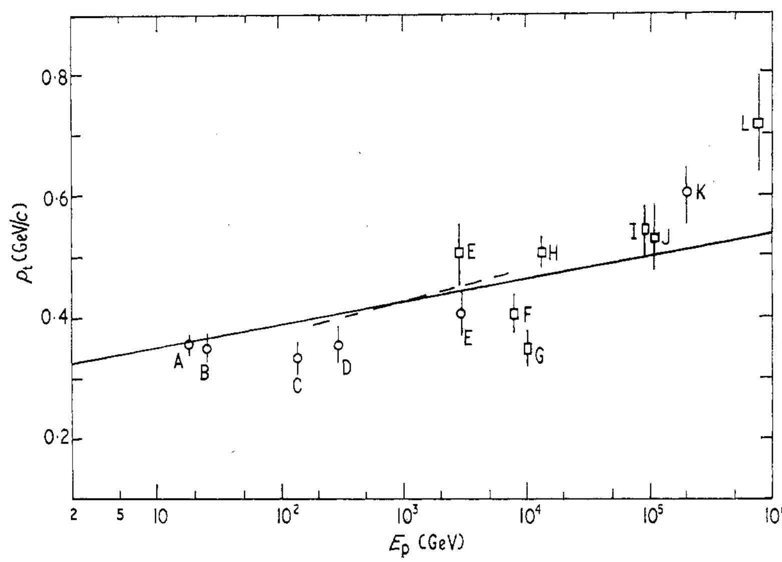

Figure 5. Summary of $\langle p_t \rangle$ values. Full line: de Beer *et al.* (1967) summary; broken line: EAS multiple muons, Rogers *et al.* (1969). ○ charged; ⊡ neutral. A, Goldsack *et al.* (1962); B, Peters (1962); C, Hansen and Fretter (1960); D, Guseva *et al.* (1962); E, Edwards *et al.* (1958); F. Minakawa *et al.* (1959); G, Akashi *et al.* (1962); H, Malhotra *et al.* (1966); I, Fowler and Perkins (1964); J, Akashi *et al.* (1966); K, present work; L, Fowler and Perkins (1964).

Also shown are the best line through a similar summary by de Beer *et al.* (1966) and that through ground level EAS data by Rogers *et al.* (1969). It is clear that taken together the data all suggest a slow increase in the mean transverse momentum with increasing interaction energy.

It can be seen that for energies above 1000 GeV the majority of data refer to neutral particles whereas below it charged particles are mainly concerned.

The value of $\langle p_t \rangle$ derived for charged particles from the present work is also shown in figure 5 at the appropriate median energy, the value being that for the distribution of Elbert *et al.* (1968). Comparison of the various estimates suggests no significant difference in $\langle p_t \rangle$ between charged and neutral particles. The difference between the Texas Lone star value and the best line not being regarded as significant, in view of the fact that the error bars shown reflect statistical errors within a single event rather than a distribution of p_t over many events as is the case for all other measurements.

The general conclusion of the present work is that the measurements support a slow increase in $\langle p_t \rangle$ with energy (for energies to at least 5×10^5 GeV) and that in the region of 2×10^5 GeV the form of the p_t distribution giving the best fit to the data is approximately $(p_t) \propto p^{\alpha}{}_t \exp(p_t/p_0)$ with $\alpha \sim 1$–$1{\cdot}5$.

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